COT 2104
Foundations of Discrete Mathematics

Practice 3 (Answers)

1.
a) { 0, 3, 6, 9, 12}

{3n | n = 0, 1, 2, 3, 4} or {x | x is a multiple of 3 $\wedge$ 0 $\leq$ x $\leq$ 12}

b) {-3, -2, -1, 0, 1, 2, 3}

{x | -3 0 $\leq$ x $\leq$ 3} we are assuming that the universe of discourse is the set of integer

or { x | x $\in$ Z $\wedge$ -3 $\leq$ x $\leq$ 3}

c) {m, n, o, p}

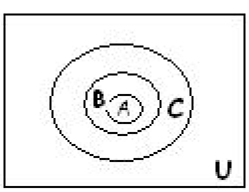
{x | x is a letter of the word monopoly other than l or y}

2. A = {2,4,6}, B = {2, 6} , C = {4, 6}, D = {4, 6, 8}

B $\subseteq$ A, C $\subseteq$ A, C $\subseteq$ D, B $\subseteq$ A, A $\subseteq$ A, B $\subseteq$ B, C $\subseteq$ C, and D $\subseteq$ D

3)
a) {∅} $\in$ { ∅ } False because the first set does not belong the second set.
b) {∅} $\in$ { { ∅ } } True because the second set has the element {∅}
c) { {∅} } $\subseteq$ { ∅, { ∅ } } True because the first set has an element that is included in the second set too.

4) Venn Diagram A $\subseteq$ B, B $\subseteq$ C



5) Two set A and B such that A $\in$ B and A $\subseteq$ B

You could find more than one answer. For example, A = ∅ and B = { ∅ }

6) Cardinality is the number of elements in a set.
a) ∅ The empty set has no elements, cardinality is 0.
b) { ∅ } This set has one element (the empty set), cardinality is 1.

c) { ∅ , { ∅ }}   This set two elements, cardinality 2.
d) { ∅ , { ∅ }, { ∅ , { ∅ }} }   This set has three elements, cardinality 3

6) Power set of a set A is the set of all subsets of A
a) ∅ The power set cannot be empty.   ∅ is not the power set of any set.
b) { ∅ , { a }}   This is the power set of {a} because the power set of every no empty set includes at least the empty set.
c) { ∅ , {a}. { ∅, a }}   Since 3 is not a power of 2, this set cannot be the power set of any set.
d) { ∅ , {a}, {b}, { a, b }}   The power set of {a, b}

8) A={a,b,c}, B={x,y}. Find A x B (The Cartesian Product)
A x B = { (a, x), (a, y), (b, x), (b, y), (c, x), (c, y)} Note: A x B ≠ B x A

9) Show that A x B ≠ B x A, when A and B are nonempty, unless A = B
Suppose A ≠ B and neither A nor B is empty

Since A ≠ B, either we can find an element x that is in A but not in B, or vice versa.

Assume that x is in A but not in B
Also, since B is not empty, there is some element y ∈ B. Then (x, y) is in A x B by definition, but is not in B x A since x ∉ B

Therefore A x B ≠ B x A

10) Translate each of the quantifications into English
a) $\exists x \in R$ ($x^3 = -1$) There is a real number whose cube is -1. True
b) $\exists x \in Z$ ((x + 1) > x) There is an integer number such that the number obtained by adding 1 to it is greater than the integer. True (every integer satisfies this statement)
c) $\forall x \in Z$ ((x – 1) ∈ Z) For every integer, the number obtained by subtracting 1 is again an integer. True
d) $\forall x \in Z$ (($x^2 \in Z$) The square of every integer is an integer. True

11) A is the set of sophomores at your school and B is the set of the students in discrete mathematics at your school.
a) The set of sophomores taking discrete mathematics in your school. $A \cap B$
b) The set of sophomores at your school who are not taking discrete mathematics.
$A \cap B^c$ (which is the same A \ B)
c) The set of students at your school who either are sophomores or are taking discrete mathematics $A \cup B$.
d) The set of students at our school who either are not sophomore or are not taking discrete mathematics
$A^c \cup B^c$

12) A= { a, b, c, d, e} and B = { a, b, c, d, e, f, g, h}
a) .A $\cap$ B = { a, b, c, d, e, f, g, h} or A $\cap$ B = B
b) A $\cup$ B = { a, b, c, d, e} or A $\cup$ B = A
c) A \ B = $\varnothing$
d) B \ A = {f, g, h}

13) Show that
a) A $\cup$ $\varnothing$ = A
A $\cup$ $\varnothing$ ={ x | x $\in$ A $\vee$ x $\in$ $\varnothing$ } = { x | x $\in$ A $\vee$ F} = {x | x $\in$ A } = A

b) A $\cap$ $\varnothing$ = $\varnothing$
A $\cap$ $\varnothing$ ={ x | x $\in$ A $\wedge$ x $\in$ $\varnothing$} = { x | x $\in$ A $\wedge$ F} = {x | F} = $\varnothing$

c) A $\cup$ A = A
A $\cup$ A = { x | x A $\vee$ x $\in$ A } = { x | x $\in$ A } = A

d) A \ $\varnothing$ = A
A \ $\varnothing$ = {x | x $\in$ A $\wedge$ x $\notin$ $\varnothing$} = { x | x $\in$ A $\wedge$ T} = {x | x $\in$ A } = A

14) A = {0, 2, 4, 6, 8, 10}, B = {0, 1, 2, 3, 4, 5, 6}, and C = {4, 5, 6, 7, 8, 9, 10}
a) A $\cap$ B $\cap$ C
A $\cap$ B $\cap$ C = { 4, 6}

b) (A $\cup$ B) $\cap$ C
(A $\cup$ B) $\cap$ C = { 4, 5, 6, 8, 10}

15) Show that

a) (A $\cap$ B) $\subseteq$ A
If x is in A $\cap$ B, then x is in A and x is in B (by definition of intersection) The result set is a subset of A

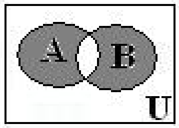
b) A \ B $\subseteq$ A
If x is in A\ B, then x is in A , but not in B(by definition of difference). A set A is a subset of itself.

16) Show that
a) (A $\cup$ B) $\subseteq$ (A $\cup$ B $\cup$ C)
Suppose x $\in$ (A $\cup$ B), then either x $\in$ A or x $\in$ B.
In either case, certainly x $\in$ A $\cup$ B $\cup$ C. This establishes the desired conclusion.

b) (A\B) \ C $\subseteq$ A \ C
Suppose that x $\in$ (A \ B) \ C, then x is A\B but not in C
Since x $\in$ A\B, x $\in$ A and x $\notin$ B
Since x $\in$ A but x $\notin$ C, we have proved that x $\in$ A\C

17) {1, 3, 5} and {1, 2, 3}
{1, 3, 5} ⊕ {1, 2, 3} = {2, 5}

18) Symmetric Difference (Venn Diagram)



19) Show that A ⊕ B = (A \ B) ∪ (B\A)

Suppose (A\B) x can be in A but not in B or
(B\A) x can be in B but not in A
Thus, an element is in A ⊕ B if and only if it is in (A\B) ∪ (B\A)

20) Determine whether the following defines an equivalence relation on the set A.

a) A is the set of all lines in the plane; a ~ b if and only if a is perpendicular to b.

Equivalence relation on a set A is a binary relation $\mathcal{R}$ on A that is reflexive, symmetric, and transitive (see pages 51-63 from your text book or the class notes (power point).

The process is:

1) **Reflexive: If a is a line, then a is not parallel to itself. The reflexive property does not hold because no line is perpendicular to itself.**
2) **Symmetric: If a ~ b a is perpendicular to be. Thus, b is perpendicular to a. Hence, b ~ a. The symmetric property holds.**
3) **Transitive: If a ~ b and b ~ c, then a is perpendicular to b and b is perpendicular to c, then a and c are parallel, not perpendicular to one another. The transitive property does not hold because a is not perpendicular to c.**

Conclusion This is not an equivalence relation because two the three properties (reflexive, symmetric, and transitive) do not hold.